LAST TIME

Vectors

$\overrightarrow{AB} = \langle a_1, a_2, a_3 \rangle$

components

$|\overrightarrow{AB}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

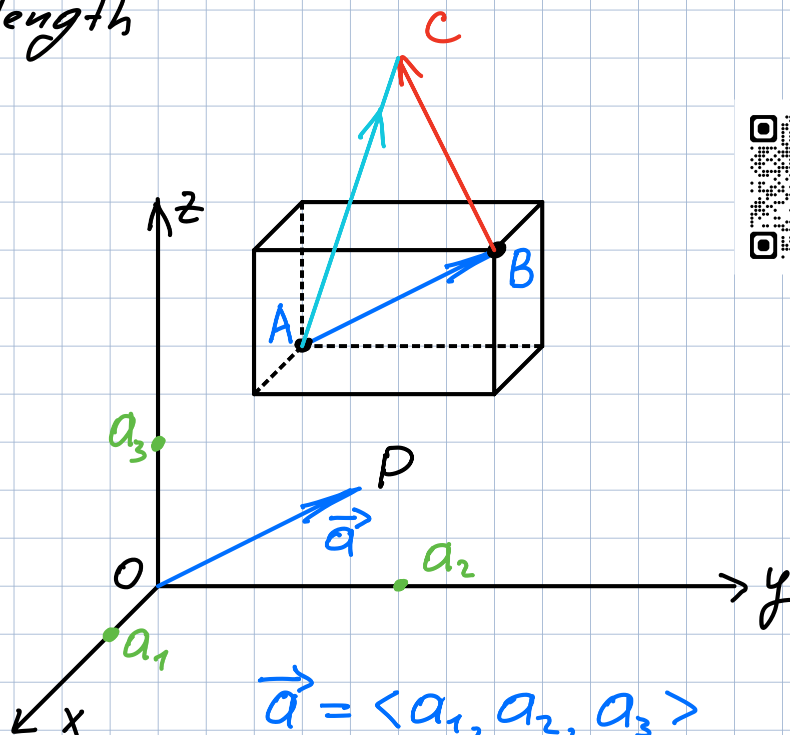
magnitude or length

$A = (x_1, y_1, z_1) \qquad B = (x_2, y_2, z_2)$

$a_1 = x_2 - x_1 \qquad a_2 = y_2 - y_1 \qquad a_3 = z_2 - z_1$

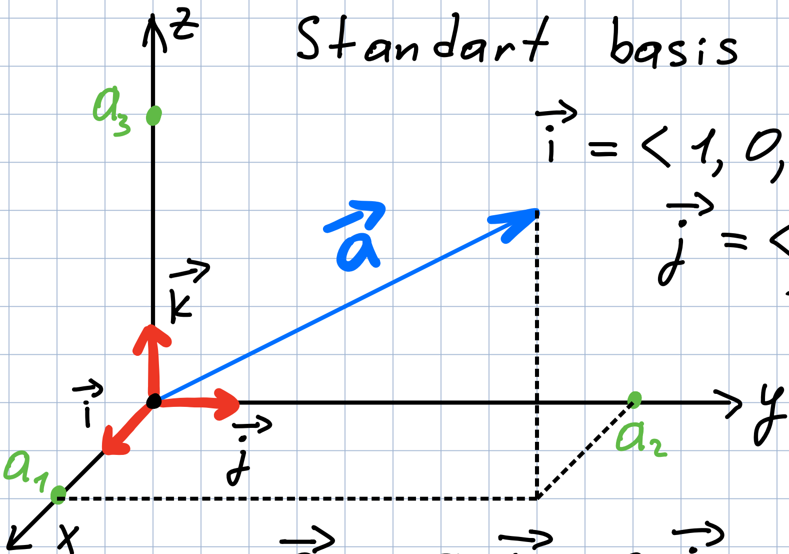
$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



$\vec{a} = \langle a_1, a_2, a_3 \rangle$

Standart basis vectors



$\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$

$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$

For $c \in \mathbb{R}$: $c\vec{a} = \langle c a_1, c a_2, c a_3 \rangle$

Unit vector:

$$\vec{u} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{|\vec{a}|}\vec{a}$$

Dot product

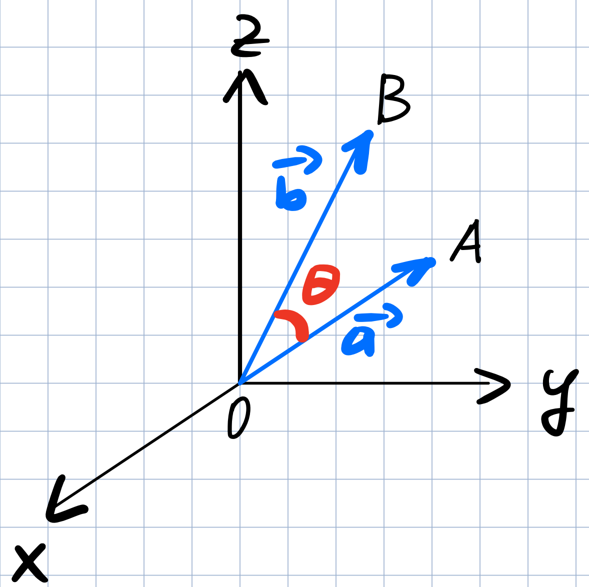
For $\vec{a} = \langle a_1, a_2, a_3 \rangle$ $\vec{b} = \langle b_1, b_2, b_3 \rangle$ two vectors,

$$\boxed{\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3}$$ the dot product

(or scalar product, or inner pr.)

Rmk: $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

Q: Does $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$ make sense?



$0 \leq \theta \leq \pi$ angle between vectors $\vec{a}, \vec{b}$

$(=$ angle between $\vec{OA}, \vec{OB})$

Thm: $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

lengths (magnitudes)

angle

Corollary:

For $\vec{a}, \vec{b}$ nonzero vectors

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

Ex: if $\vec{a}, \vec{b}$ have length 1, 3 and $\theta = \frac{\pi}{3}$

then $\vec{a} \cdot \vec{b} = 1 \cdot 3 \cdot \cos\frac{\pi}{3} = 1 \cdot 3 \cdot \frac{1}{2} = \frac{3}{2}$

Ex: $\vec{a} = \langle 1, 1, 1 \rangle$ find θ.

$\vec{b} = \langle 1, 2, 3 \rangle$

Solution: $|\vec{a}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$ $\qquad \vec{a} \cdot \vec{b} = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 = 6$

$|\vec{b}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$ $\qquad \Rightarrow \cos\theta = \frac{6}{\sqrt{14}\sqrt{3}} = \frac{6}{\sqrt{42}}$

$\Rightarrow \theta = \arccos\frac{6}{\sqrt{42}} \approx 22^\circ$

- Non zero vectors are called orthogonal (or perpendicular) if the angle $\theta = \frac{\pi}{2}$ $\iff$ $\vec{a} \cdot \vec{b} = 0$

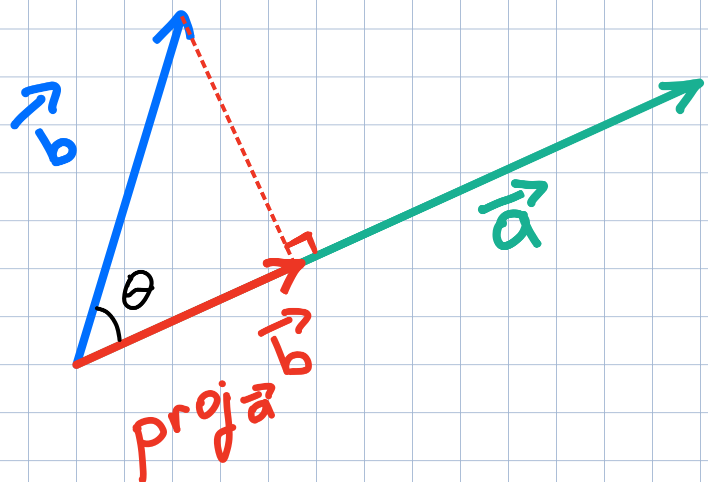
Ex: $\vec{a} = \langle -1, 2, 5 \rangle$

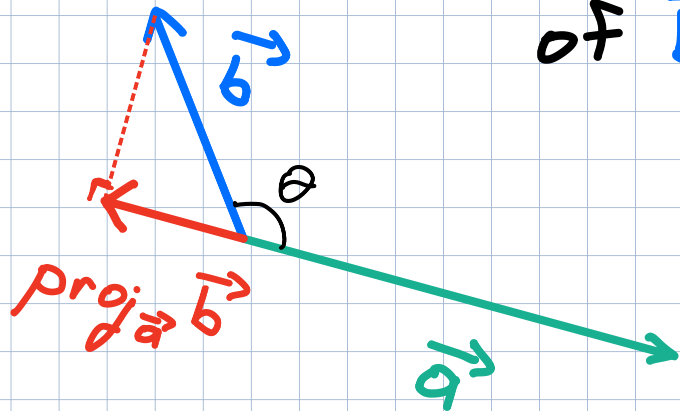
$\vec{b} = \langle 1, -2, 1 \rangle$

$\vec{a} \cdot \vec{b} = (-1)1 + 2(-2) + 5(1) = 0$

$\Rightarrow$ $\vec{a}$ and $\vec{b}$ are orthogonal

Projections





vector projection of $\vec{b}$ onto $\vec{a}$

the scalar projection of $\vec{b}$ onto $\vec{a}$ (also called the component of $\vec{b}$ along $\vec{a}$)

is the signed magnitude of the vector projection

$$\text{comp}_{\vec{a}}\vec{b} = |\vec{b}|\cos\theta$$

Recall

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$\text{comp}_{\vec{a}} \vec{b} = |\vec{b}| \cos\theta$$

$$\Rightarrow \quad \boxed{\begin{gathered} \text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \\ \text{and} \\ \text{proj}_{\vec{a}} \vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a} \end{gathered}}$$

Ex: Find the scalar projection and vector projection of $\vec{b} = \langle 1, 1, 2 \rangle$ onto $\vec{a} = \langle -2, 3, 1 \rangle$

Solution: 1) Find $|\vec{a}|$ and $\vec{a} \cdot \vec{b}$

$$|\vec{a}| = \sqrt{(-2)^2 + 3^2 + 1^2} = \sqrt{14}$$

$$\vec{a} \cdot \vec{b} = (-2)1 + (3)1 + 1(2) = 3$$

2) $$\text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{3}{\sqrt{14}}$$

3) The vector projection is the scalar projection times the unit vector in the direction of $\vec{a}$:

$$\text{proj}_{\vec{a}} \vec{b} = \frac{3}{\sqrt{14}} \frac{\vec{a}}{|\vec{a}|} = \frac{3}{14} \vec{a} = \left\langle -\frac{3}{7}, \frac{9}{14}, \frac{3}{14} \right\rangle$$